

**Fifth Semester B.E. Degree Examination, Dec. 07 / Jan. 08**  
**Modern Control Theory**

Time: 3 hrs.

Max. Marks: 100

**Note : Answer any FIVE full questions.**

- 1 a. **Define** controller. Explain PD, PI and PID controllers. What are the advantages of PID controller? (06 Marks)
- b. **Define** the concept of i) State ii) State space iii) State variables. (06 Marks)
- c. For the network shown in fig.Q1(c), choosing  $i_1(t) = x_1(t)$  and  $i_2(t) = x_2(t)$  as state variables, obtain the state equation and output equation in vector matrix form. (08 Marks)

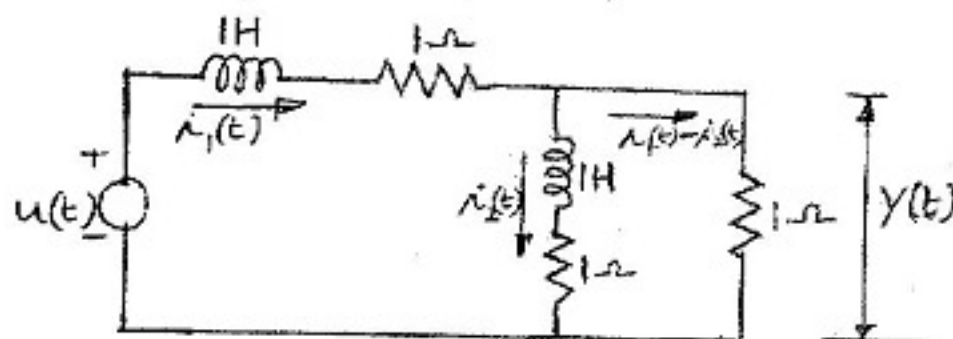


Fig.Q1(c)

- 2 a. What is state transition matrix? State and prove the properties of state transition matrix. (06 Marks)

- b. Obtain the model matrix for the matrix given below:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 2 \\ -12 & -7 & -6 \end{bmatrix}$$

(04 Marks)

- c. Obtain the state transition matrix using:

- i) Laplace transformation method  
 ii) Cayley - Hamilton method for the system described by the matrix given below

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

(10 Marks)

- 3 a. Consider a system having state model:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \end{bmatrix} u \quad \text{and} \quad Y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \quad \text{with } D = 0. \quad \text{Obtain its transfer function.} \quad (08 \text{ Marks})$$

- b. Determine the complete time response of the system given by:

$$\dot{X}(t) = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} X(t), \quad \text{where } X(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \text{and} \quad Y(t) = \begin{bmatrix} 1 & -1 \end{bmatrix} X(t). \quad (12 \text{ Marks})$$

- 4 a. Mention the differences between state space techniques and classical approach. (06 Marks)  
 b. Obtain the state model in phase variable form and write the block diagram for the system represented by,

$$\frac{d^3 y}{dt^3} + 6 \frac{d^2 y}{dt^2} + 11 \frac{dy}{dt} + 10y = 3u(t). \quad (06 \text{ Marks})$$

- c. For the following transfer function obtain the state model in canonical form:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{2}{s^3 + 6s^2 + 11s + 6}. \quad (08 \text{ Marks})$$

- 5 a. Define the concepts of controllability and observability. Explain the principle of duality between controllability and observability. (10 Marks)  
 b. Determine the controllability and observability using Kalman's test for the system described by,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \text{and} \quad C = [10 \ 0 \ 0]. \quad (10 \text{ Marks})$$

- 6 a. Consider the system defined by  $\dot{X} = Ax + Bu$ , where  $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}$   $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ . By

using state feedback control  $u = -Kx$ , it is desired to have the closed loop poles at  $s = -2 \pm j4$  and  $s = -10$ . Determine the state feedback gain matrix "K" by any one method. (08 Marks)

- b. Consider the system  $\dot{X} = Ax + Bu$  and  $Y = Cx$ , where  $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$   $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$  and

$C = [1 \ 0 \ 0]$ . Determine the observer gain matrix by the use of:

- i) The direct substitution method  
 ii) Ackermann's formula.

Assume that the desired Eigen values of the observer gain matrix are  $\mu_1 = -2 + j3.4641$  and  $\mu_2 = -2 - j3.4641$ ,  $\mu_3 = -5$ . (12 Marks)

- 7 a. Explain any three non-linearities in control systems. (06 Marks)  
 b. What is a singular point? Explain the different types of singular points in a non-linear control system based on the location of Eigen values of the system. (08 Marks)  
 c. Explain the construction of a phase trajectory by Delta method. (06 Marks)

- 8 a. Explain Liapunov's stability criterion and also explain Liapunov's theorems on:  
 i) Asymptotic stability  
 ii) Asymptotic stability in the large  
 iii) Instability. (10 Marks)

- b. A system is described by the following equation:

$$\dot{X} = AX, \quad \text{where} \quad A = \begin{bmatrix} -1 & -2 \\ 1 & -4 \end{bmatrix}.$$

Assuming matrix Q to be the identity matrix, solve for matrix P and comment on the stability of the system using the equation  $A^T P + PA = -Q$ . (10 Marks)